



MATH 415 PL1

Applied Linear Algebra

Lectures, Labs, Discussions.

- ➔ Lecture: Asynchronous online (two hours a week)
- ➔ Labs: Fridays synchronous online on Zoom
 - 📶 Section PY1, 10am, Zoom ID: 929 1108 1470, Password: 7r635t
 - 📶 Section PY2, 2pm, Zoom ID: 975 0396 1600, Password: 5l364b
 - 📶 Section PY3, 3pm, Zoom ID: 930 7649 6529, Password: 7u179c
- ➔ Discussion Sections: synchronous online, various times Tuesdays and Thursdays (Sections PDA, PDB, PDC, PDD, PDE, PDF, PDG, PDH, PDJ, PDS, PDT)
- ➔ TA office hours: TBA
- ➔ Instructor office hours: TBA

Instructors & TAs.

- ➔ Lecture instructor: Philipp Hieronymi (phierony@illinois.edu)
- ➔ Lab instructor: Philipp Hieronymi, Mariana Silva (mfsilva@illinois.edu)
- ➔ TAs: Eion Blanchard (eionmb2@illinois.edu), Elliot Kaplan (eakapla2@illinois.edu), Elizabeth Tatum (etatum2@illinois.edu)

Introduction.

This is a first course in linear algebra. This covers basic definitions and algorithms of the subject needed in the higher level (engineering, science and economics) courses and more sophisticated mathematical techniques such as the Singular Value Decomposition.

In this course you learn the mathematical theory and how to implement it in Python. You will discover many of the striking modern applications of linear algebra, such as Google's PageRank algorithm, image and audio compression schemes such as JPEG and MP3, automatic face recognition and other data science and machine learning algorithms.

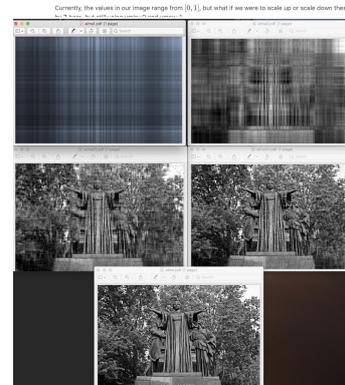
The course covers the same mathematical theory as MATH 415, but adds a focus on the computational and large data aspect of linear algebra through the lab sessions. This is a pilot for a future course MATH 257 - Linear Algebra with Computational Applications.

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What happens when you plot the transposition of the image?
In [28]: plt.figure()
plt.imshow(lisoon1a.T, cmap="gray", vmin=0, vmax=1)
nextipynb.lib.image.Image at 0x7f022063a000
4
Out[28]:
0 50 100 150 200 250
150
100
50
0
Can you invert the image? What happens if you subtract 1 - each value in the array?
In [29]: lisoon1a = 1 - x for x in lisoon1a
plt.figure()
plt.imshow(lisoon1a, cmap="gray", vmin=0, vmax=1)
nextipynb.lib.image.Image at 0x7f021f7c1a00
4
Out[29]:
0 50 100 150 200 250
150
100
50
0

```

Changing Image Values



Technical equipment. This course will be conducted entirely online. As such, each student will be assumed throughout the semester to have the necessary technical equipment to participate in course activities:

- 🔊 a computer/laptop/tablet with a webcam and a microphone,
- 🔊 a stable internet access, sufficient bandwidth and data allowance for using a webcam on Zoom.

Please contact the Student Assistance Center (helpdean@illinois.edu) immediately if you are missing any of required technology.

Other Linear Algebra courses. Be aware that course credit is not given for both MATH 415 and any of MATH 125, MATH 225, MATH 227, MATH 416 or ASRM 406. Any enrollment related questions should be sent to mathadvising@illinois.edu.

Three disclaimers.

- ⚠ This is not a course that only teaches you how to compute stuff. Computer will always be faster. Modern applications of linear algebra require a sophisticated understanding of theory and methods of linear algebra, and learning these is the purpose of this course. Some of it might look like *abstract* linear algebra. However, through the applications we cover in the labs, you realizes that this indeed is *applied* linear algebra.
- ⚠ If you already know some linear algebra, this course might look easy at the beginning. Don't be fooled into thinking it will stay like that. Even the material familiar to you will be covered in more depth here. Furthermore, the exams will require a deeper understanding of the concepts you already know something about. So it is a good idea to take this course seriously from the beginning.
- ⚠ MATH 415 Section PL1 is a pilot for the new course MATH 257. A significant part of the material for this new course is - not surprisingly - new. So if you find a typo or an error in any part of this course, please let us know by sending an email to the instructors. We appreciate your help, and are also happy to hear any further comments or suggestions. Thank you!

Learn@Illinois. This course has a page on Learn@Illinois:

<https://learn.illinois.edu/course/view.php?id=50925>

All material will be available there. **You can check all your grades on this website.** Please note that if you have just registered for the course, you will automatically be given access to the Learn@Illinois website within a few hours. Only if you do not have access to the course site 48 hours after registering, contact your instructor.

Setup. This course consists of two hours per week of lecture, one hour per week of computing lab, and one hour per of week of active learning (paper-based) discussion sections.

Discussion section. Discussion sections are held synchronously online on Zoom on Tuesdays and Thursdays:

- 📶 Section PDA, Tuesdays 10am, Zoom ID: 978 3125 6749, Password: 0r084y
- 📶 Section PDB, Tuesdays 11am, Zoom ID: 945 2838 9364, Password: 7d925f
- 📶 Section PDC, Tuesdays 2pm, Zoom ID: 916 5495 4391, Password: 9t565v

- 📶 Section PDD, Tuesdays 3pm, Zoom ID: 913 0011 6200, Password: 0s954w
- 📶 Section PDE, Thursdays 10am, Zoom ID: 952 5807 2714, Password: 2g189v
- 📶 Section PDF, Thursdays 11am, Zoom ID: 998 0609 9364, Password: 3a663n
- 📶 Section PDG, Thursdays 2pm, Zoom ID: 956 9253 2482, Password: 2y439a
- 📶 Section PDH, Thursdays 3pm, Zoom ID: 976 0310 8549, Password: 2t957c
- 📶 Section PDJ, Thursdays 9am, Zoom ID: 929 0259 1695, Password: 5w445b
- 📶 Section PDS, Tuesdays 1pm, Zoom ID: 977 0766 9194, Password: 1b394d
- 📶 Section PDT, Tuesdays 9am, Zoom ID: 990 1899 7014, Password: 9l092k

Only attend the discussion section you are signed up for.

During the discussion sections, TAs will distribute worksheets to be completed collaboratively in small groups. At the end of the period, each group will submit one worksheet. Complete solutions to the worksheet will be posted afterwards on Learn@Illinois.

Attendance will be taken. You will be given a password at the beginning of each discussion section and you will have 15 minutes to mark yourself present on Learn@Illinois. Note that it is not enough to just be present. You have to be actively working with your group, and the worksheet submitted by your group must show that your group put in the necessary effort. If this is not the case, we will not consider you present and will not receive points for participation.

Unless there are special circumstances or you are told otherwise by the TA, we expect you to have your video and your audio active through the whole Zoom meeting.

Computational Lab. Labs are held synchronously online on Zoom on Fridays:

- 📶 Section PY1, 10am, Zoom ID: 929 1108 1470, Password: 7r635t
- 📶 Section PY2, 2pm, Zoom ID: 975 0396 1600, Password: 5l364b
- 📶 Section PY3, 3pm, Zoom ID: 930 7649 6529, Password: 7u179c

Please only attend the lab sections you are signed up for.

In the labs you will use computational tools in Python to solve linear algebra problems in real world applications in science and engineering. You will be working in small groups on a Python worksheet together. For students who never used Python before, we will have a Python tutorial in the first lab session. Please bring your own device to the lab sessions.

We will be using CoCalc. Please create a CoCalc account at <https://cocalc.com/>, and bring your login name to the first lab.

Attendance will be taken. You will be given a password at the beginning of each lab and you will have 15 minutes to mark yourself present on Learn@Illinois. Note that it is not enough to just be present. You have to be actively working with your group on the project. If this is not the case, we will not consider you present.

Textbook. We will post extensive lecture notes for all lectures and practice problems online. For many students these notes are enough. If you still want to buy/download a book, here are three options:

- 📖 Philip N. Klein, *Coding the Matrix: Linear Algebra through Applications to Computer Science*, first edition, Newtonian Press
- 📖 David Cherney, Tom Denton, Rohit Thomas, Andrew Waldron, *Linear Algebra*, <https://www.math.ucdavis.edu/~linear/>
- 📖 Gilbert Strang, *Linear Algebra and its Applications*, fourth edition, Cengage.

You are not required to buy any of these textbooks.

Slides. Lecture notes have been on Learn@Illinois. An interactive version with fill-in boxes is also available.

Videos. We will post module videos on the Learn@Illinois page of this course. An interactive version of the slides with fill-in boxes is also available. If you would like to use this feature, print out the fill-in slides and fill them out on your own or while watching the videos.

There are many other great (free) videos about linear algebra. Here are some we recommend as an addition (not a substitute) for the lecture videos.

- 🎥 *Essence of Linear Algebra* by 3Blue1Brown, on Youtube
- 🎥 MIT lectures by Gilbert Strang, MIT Open Courseware
- 🎥 *Coding the Matrix* videos by Philip Klein, on Youtube

Online homework. Each module comes with two sets of homework, both available through the Learn@Illinois website.

The Checkpoint quizzes consist of two to three easy conceptual questions. We recommend that you take the quiz immediately after watching the module video. These quizzes make sure that you have watched the video and have at least a basic understanding of concepts covered in the video. *Note that you only have one attempt for each quiz.*

The PrairieLearn homework associated with each module focuses on the computations and algorithms covered in the module. In this homework you will have to do the computation we did in the video by yourself. See below for more information on PrairieLearn.

Weekly assignment schedule. Weekly assignments due dates are as follows:

- 🕒 Checkpoint Quizzes (covering modules from previous week), due Mondays 11:59PM
- 🕒 PrairieLearn (non-lab) Homework (covering modules from previous week), due Mondays 11:59PM
- 🕒 PrairieLearn lab Homework (covering lab from previous week), due Thursdays 11:59PM.

For example, checkpoint quizzes 1-4 are due on Monday August 31st (Monday of Week 2). PrairieLearn Week 1 homework is due on Monday August 31st (Monday of Week 2).

Netiquette. Since this is an online course, please be respectful of your fellow classmates and teaching staff in all online communications. Fostering a helpful learning environment requires everyone's cooperation. Remember that forum posts are visible to all students and staff in the course (around 300 people). So please double-check your posts before submitting them.

PrairieLearn. We will use PrairieLearn for homework. You have to access PrairieLearn through the Learn@Illinois website.

Homework will be due on Mondays at 11:59PM. The first homework is due on Monday 08/31 (see Week 1 tab). The PrairieLearn homework will focus on computations, while the worksheets in the discussion section will focus more on conceptual problems.

How points are given on PrairieLearn. PrairieLearn places emphasis on mastery. The idea is to keep doing questions until you master the underlying concept or method. Once you do, you should be able to answer these questions very quickly.

The way this works in PrairieLearn is that each question has a value, a point total, and a point maximum. If you answer a question correctly, two things happen:

- ▶ The point total increases by the value, until you reach the point maximum.
- ▶ The value increases.

If you answer a question incorrectly, one thing happens:

- ▶ The value goes back to what it was originally.

This system rewards repeated correct answers, which tend to demonstrate mastery. There is no penalty (other than resetting the value) for answering a question incorrectly, so don't be afraid to submit an answer. Similarly, don't be afraid to keep doing a question after you reach the point maximum - your point total will never go down!

Credit. There is no need to "submit" your homework. The system will record whatever your score is at that time. However, you'll note the following line at the top of your screen:

Available credit: 110% until 11:59PM, Mon, 08/31

What this means is that if you reach 100% prior to 11:59AM on that Monday - i.e., complete the homework early - you will receive an extra 10% bonus. You will see this reflected in your score (the instant you reach 100%, it will jump to 110%).

If you click on the "?" just to the right of the line about available credit, you'll see all the dates associated with this homework. In particular, it says:

- ▶ you can receive 110% until 11:59PM, Monday, August 31st,
- ▶ you can receive 80% until 11:59PM, Wednesday, December 9th.

Note that your score will never go down. For example, if you achieve 90% by 11:59PM on Monday, August 31st, you won't be able to increase your score after that time, but you won't be penalized for not reaching 100% - your score will remain 90% forever. On the other hand, if you achieve only 70% by 11:59PM on Monday, August

31st, you will be able to increase your score after that time (to a maximum of 80%).

Please note that your overall PrairieLearn score is capped at 100%. So we even if you score 110% on every assignment, you will only receive 100% overall.

Typos/Errors. If you believe there is a typo or an error in a question, or if you believe your answer was graded incorrectly, please take a screenshot and send an email to your instructor. We have access to all of your submissions and can easily check to see what, if anything, went wrong.

CampusWire. All announcements will be posted on CampusWire at

<https://campuswire.com/p/G10BC552A>.

Please make sure you are signed up for CampusWire (PIN: 0758). Questions about the course material, or the organization of the course, that potentially are of interest to everyone in the class, should all be posted on CampusWire.

If you have a private question (for example about your grade or because you have to miss an exam), please contact your instructor via email. When posting on CampusWire, please use the subject line wisely. For example, if you ask something about matrix multiplication in Lecture notes 5, write “Lecture notes 5 - Matrix multiplication” and not just “Question about matrices”. In addition, please post to the entire class whenever this is appropriate. No question will ever be held against you.

When you send an email to one of the instructors, please clearly write your name somewhere and please start the subject line with “MATH 415”. This will make sure that your mail isn’t lost.

Exams. There will be three midterm exams, each about 50 minutes long, and a three hour final exam.

🎓 **Midterm 1: Friday, September 18**

🎓 **Midterm 2: Friday, October 16**

🎓 **Midterm 3: Friday, November 13**

🎓 **Final: TBA.**

The midterms will take place during the usual lab times and will be administered by CBTF (see below). Thus:

🎓 If you are registered for Lab PY1, you will take your exam at 10am.

🎓 If you are registered for Lab PY2, you will take your exam at 2pm.

🎓 If you are registered for Lab PY3, you will take your exam at 3pm.

There will be no make-up exams. Instead, if you miss an exam and have a valid excuse, we will mark the exam as ‘excused’. An ‘excused’ exam means that this exam will not be taken into account in the computation of your grade. **Valid excuses must be documented** and must be reported to your instructor immediately.

CBTF. This course uses the College of Engineering Computer-Based Testing Facility (CBTF) for its quizzes and exams:

<https://cbtf.engr.illinois.edu>.

The policies of the CBTF are the policies of this course, and academic integrity infractions related to the CBTF are infractions in this course.

If you have accommodations identified by the Division of Rehabilitation-Education Services (DRES) for exams, please email your Letter of Accommodation (LOA) to CBTF Manager Carleen Sacris at sacris1@illinois.edu before you make your first exam reservation.

Any problem with testing in the CBTF must be reported to CBTF staff at the time the problem occurs. If you do not inform a proctor of a problem during the test, then you forfeit all rights to redress.

Cheating. No books, notes, calculators, cheat sheets or electronic devices are allowed during the exams. We take cheating very seriously! A more detailed description of the University policy on cheating and plagiarism may be found in the following link:

<http://www.las.illinois.edu/students/integrity/>

Grading. The course grade will be the average of your homework, worksheets, midterm exams, and final exam grades, weighted as follows:

- ▶ 5% discussion section attendance (the two lowest scores will be dropped)
- ▶ 5% lab attendance (the two lowest scores will be dropped)
- ▶ 8% PrairieLearn (non-lab) homework (the two lowest scores will be dropped)
- ▶ 5% PrairieLearn lab homework (the two lowest scores will be dropped)
- ▶ 2% Check point quizzes (the ten lowest scores will be dropped)
- ▶ 17% each Midterm exam (total 51%)
- ▶ 24% final exam

In addition: If your final exam score is higher than one of your midterm scores, then we will replace your lowest midterm score by your final exam score.

If you miss one midterm (**and have a valid excuse**), we will use the average of the two other midterms and the final exam as the score for the midterm you missed. We then apply the above calculation (including the potential replacement of your lowest midterm score).

If you miss more than one midterm, please contact your instructor.

Letter grades will be assigned according to (this is for the percentage, not for the absolute score!):

- ▶ 100.00 % - 98.00 % → A+
- ▶ 97.99 % - 93.00 % → A
- ▶ 92.99% - 90.00% → A-
- ▶ 89.99 % -87.00 % → B+
- ▶ 86.99 % - 83.00 % → B
- ▶ 82.99 % - 80.00 % → B-
- ▶ 79.99 % - 77.00 % → C+

- ▶ 76.99 % - 73.00 % → C
- ▶ 72.99 % - 70.00 % → C-
- ▶ 69.99 % - 67.00 % → D+
- ▶ 66.99 % - 55.00 % → D
- ▶ 54.99 % - 0.00 % → F

Usually around 30% of the students get an A letter grade (including +/-) and around 70% score a B letter grade or higher. The median score is usually between 83% and 84%. The average GPA of this course over the last few years has been around 3.0. This will also be the case this semester.

We will renormalize each of the midterms and final exam such that the distribution of letter grades coincides with this historic distribution of the letter grades for MATH 415. No further curve will be applied at the end of the course.

There will be **no extra credit**. So make sure to work hard for every midterm!

Please check each week that your score was entered correctly on Learn@Illinois. With so many students it can happen that your grade is entered incorrectly. If, after an exam or a quiz, you find an error in the grading of your exam, please see us *immediately* before or after class/discussion section or during our office hours. It can always happen that we made a mistake while grading your exam, so we always encourage you to see us if you think that happened. Rescoring requests will only be considered within a week after an exam or quiz is handed back. So don't wait! With 1000 students there are always many cases where students are close (sometimes even very close) to the next letter grade, and at the end of the semester make the case that they should receive higher grades. Unfortunately, in almost all cases we can not grant the request without being unfair to other students—even if we would like to!

Course Calendar

Date	Week	Lecture	Lab	Topic
08/24	1	1		Introduction to linear systems, Matrices
08/26	1	2		Echelon form of matrices, Gaussian Elimination
08/28	1		1	Python tutorial
08/31	2	3		Linear combinations, Matrix vector multiplication
09/02	2	4		Matrix multiplication, Properties of matrix multiplication
09/04	2		2	Working with vectors
09/07	3			Labor day - No class
09/09	3	5		Elementary matrices, Inverses of matrices
09/11	3		3	Matrix operations
09/14	4	6		Computing an inverse, LU decomposition
09/16	4	7		Solving linear systems using LU, Inner products
09/18	4			Midterm 1
09/21	5	8		Subspaces of \mathbb{R}^n , Column spaces and nullspaces
09/23	5	9		Abstract vector spaces, Linear independence
09/25	5		4	Solving systems of linear equations
09/28	6	10		Basis and dimension
09/30	6	11		The four fundamental subspaces, Orthogonal complements
10/02	6		5	Graphs and Algebraic Graph Theory
10/05	7	12		Coordinates, Orthonormal bases
10/07	7	13		Linear transformations, Coordinate matrix
10/09	7		6	Data compression
10/12	8	14		Determinants, Cofactor expansion
10/14	8	15		Eigenvectors and Eigenvalues
10/15	8			Midterm 2
10/19	9	16		Computing eigenvectors and eigenvalues
10/21	9	17		Properties of eigenvectors, Markov matrices
10/23	9		7	Markov Chains
10/26	10	18		Diagonalization, Powers of matrices
10/28	10	19		Matrix Exponential, Orthogonal projection onto lines
10/30	10		8	Dynamical Systems
11/02	11	20		Orthogonal projection onto subspaces, Least squares solutions
11/04	11	21		Linear Regression
11/06	11		9	Linear Regression
11/09	12	22		Gram-Schmidt process, QR decomposition
11/11	12	23		Spectral Theorem
11/12	12			Midterm 3
11/17	13	24		SVD, Low rank approximations
11/19	13	25		Pseudo-Inverse, Least squares solutions via SVD
11/21	13		10	SVD and applications
11/30	15	26		Principal Component Analysis
12/02	15	27		Review complex numbers, Complex linear algebra
12/04	15		11	Principal Component Analysis
12/07	16	28		Leeway
12/09	16	29		Review